Supplemental Material: Hierarchical Fuzzy-Cluster-Aware Grid Layout for Large-Scale Data

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APPENDIX A: GRID VISUALIZATIONS IN THE REQUIREMENT ANALYSIS

In Tab. 1, we list all the grid visualizations in the toolkits used by the experts and their disadvantages. In summary, all these grid visualizations are less effective in analyzing fuzzy clusters, and most of them cannot support the analysis of large-scale datasets. This motivated us to develop a new tool, which can support hierarchical exploration and analysis of fuzzy clusters in large-scale datasets.

APPENDIX B: DYNAMIC WEIGHT ADJUSTMENT

Since the dynamic weight adjustment is utilized more than once in our method, we give a general example to demonstrate how it works. Suppose that we are optimizing the following function:

$$\min_{O} \mu_1 O_1 + \mu_2 O_2 + \mu_3 O_3.$$

Here, O_1 , O_2 , and O_3 are three objectives to optimize. Weights μ_1 , μ_2 , and μ_3 balance the three terms and are determined with the multi-task learning method [6]. The key idea of the multi-task learning method is dynamically increasing the weights of the terms that are not well-optimized so that they can be further improved. The degree of optimization is measured by:

$$\Delta(O_i, \theta) = (O_i(\theta) - O_i^{\min}) / (O_i^{\max} - O_i^{\min}) \text{ for } i = 1, 2, 3.$$

 O_i^{max} and O_i^{min} are the maximum and minimum values of the *i*-th objective, respectively. Since determining the exact values of O_i^{max} and O_i^{min} is challenging, we used two simple strategies to approximate their values. In particular, O_i^{min} is approximated as the value of the *i*-th objective when the *i*-th objective is optimized only, and O_i^{max} is approximated as the maximum value of the *i*-th objective when the other ones are optimized:

$$O_i^{\min} = \min_{\theta} O_i,$$

$$O_i^{\max} = \max_{j \neq i} O_i(\theta_j) \text{ where } \theta_j = \arg\min_{\theta} O_j.$$
(1)

Initially, μ_1 , μ_2 , and μ_3 are set as 1 and $\Delta(O_1, \theta)$, $\Delta(O_2, \theta)$, and $\Delta(O_3, \theta)$ are calculated. Then, μ_1 , μ_2 , and μ_3 are updated:

$$\mu_i = \mu_i + \mathbb{I}_i \cdot 2^{-k}, \text{ for } i = 1, 2, 3.$$
⁽²⁾

 $\mathbb{I}_i = 1$ if $\Delta(O_i, \theta)$ is the largest among $\Delta(O_1, \theta)$, $\Delta(O_2, \theta)$, and $\Delta(O_3, \theta)$. $\mathbb{I}_i = 0$ if $\Delta(O_i, \theta)$ is the middle value, and $\Delta(O_i, \theta) = -1$ if $\Delta(O_i, \theta)$ is the smallest. *k* is the current iteration number. Once μ_1, μ_2 , and μ_3 are updated, this process is repeated again until the layout process converges.

APPENDIX C: SIMULATED ZOOM-IN OPERATIONS IN THE QUANTITATIVE EXPERIMENTS

In the quantitative experiments, the three baselines (Zhou *et al.*'s method [8], DendroMap [5], and LAS [4]) and our method use different zoom-in operations.

Zhou *et al.*'s **method**, **LAS**, **and our method**. For Zhou *et al.*'s method, LAS, and our method, as users can freely select areas of any positions and sizes in the grid layouts for zooming, we use the same zoom-in operations for them. Specifically, we simulated several zoom-in operations by randomly selecting areas in the grid layouts (*e.g.*, Fig. 1(a)).

DendroMap. For DendroMap, all samples are organized into a static binary hierarchy. At each time, several nodes in the binary hierarchy are displayed as a treemap. During the exploration, users can only select one treemap node for zoom-in. Fig. 1(b) shows an example of such a zoom-in operation.

APPENDIX D: FULL QUANTITATIVE EXPERIMENT RESULTS

Experiment detials. For each dataset, we evaluate the baselines and our method with different grid sizes. For the comparison with Zhou *et al.*'s method and LAS, we use sizes of 30×30 , 40×40 , and 50×50 for experiments. For the comparison with DendroMap, its grid sizes cannot be specified directly. Instead, only the sizes of the images can be specified to change the grid sizes. Therefore, we set the sizes of the images as 30px, 20px, and 16px in a display with a resolution of around 1700×1060 . These three image sizes correspond to grid sizes of around 30×30 , 40×40 , and 50×50 .

Manuscript received xx xxx. 201x; accepted xx xxx. 201x. Date of Publication xx xxx. 201x; date of current version xx xxx. 201x. For information on obtaining reprints of this article, please send e-mail to: reprints@ieee.org. Digital Object Identifier: xx.xxx/TVCG.201x.xxxxxx

Tool names	Grin theutabileitons	large Ascal yz iag asets	fu Azjalynintg rs
FiftyOne [7]		×	×
Lightly [2]	Explore the samples in your dataset Compare with initial tag Compare wit	×	×
Know Your Data [1]		×	×
TensorBoard [3]		×	×
Zhou's method [8]		<i>J</i>	×

Table 1: The grid visualizations in the toolkits used by the experts.

For the comparison between our method, Zhou *et al.*'s method, and LAS, we simulated 150 zoom-in operations for each dataset and each grid size in both methods. For the comparison between our method and DendroMap, as the zoom-in operations are restricted to the treemap nodes, the number of total possible zoom-in operations is limited. Therefore, we first simulated 30 zoom-in operations for DendroMap. Then, for each zoom-in operation, we randomly select five areas with the same grid size in our method. As a result, a total of 150 grid layouts were generated for our method.

Results. For the comparison between our method and Zhou *et al.*'s method, Tabs. 2 to 4 show the comparison of compactness, convexity, proximity, stability, and ambiguity on 3 datasets. For the comparison between our method and LAS, Tabs. 5 to 7 show the results of both methods on 3 datasets. For the comparison between our method and DendroMap, Tabs. 8 to 10 show the results of both methods on 3 datasets.



Fig. 1: Zoom-in operation examples of (a) Zhou *et al.*'s method, LAS, and our method; (b) DendroMap. Note that each color in (a) corresponds to a cluster in the input hierarchy of the dataset, while a treemap node (rectangle) in (b) corresponds to a node in the static binary hierarchy organized by DendroMap.

Conv. type	Size	Method	Comp. \downarrow	Conv. ↑	Prox. ↑	Stab-shape \uparrow	Stab-position \downarrow	Stab-sample \downarrow	Ambi. \downarrow
	30×30	Zhou <i>et al.</i> 's method Ours	0.021 0.021	0.997 0.996	598.3 618.0	0.73 0.92	0.092 0.063	181.2 0.3	1.16 0.97
ratio	40×40	Zhou <i>et al.</i> 's method Ours	0.016 0.016	0.997 0.997	527.6 566.9	0.77 0.94	0.078 0.061	563.9 0.7	1.98 1.61
	50×50	Zhou <i>et al.</i> 's method Ours	0.016 0.016	0.998 0.998	456.9 518.9	0.76 0.95	0.088 0.052	1475.7 1.7	3.30 2.66
Perimeter ratio	30×30	Zhou <i>et al.</i> 's method Ours	0.022 0.023	0.930 0.965	585.9 624.0	0.64 0.89	0.121 0.081	196.0 0.7	1.23 0.94
	40×40	Zhou <i>et al.</i> 's method Ours	0.018 0.018	0.928 0.964	513.1 562.9	0.68 0.90	0.097 0.075	607.0 1.2	2.04 1.56
	50×50	Zhou <i>et al.</i> 's method Ours	0.017 0.017	0.927 0.970	449.5 515.0	0.65 0.91	0.105 0.074	1557.9 3.4	3.28 2.80

Table 2: Comparison of Zhou *et al.*'s method and ours in CIFAR-100. \uparrow (\downarrow) indicates the higher (lower) is better.

Table 3: Comparison of Zhou *et al.*'s method and ours in iNat2021-mini. $\uparrow(\downarrow)$ indicates the higher (lower) is better.

Conv. type	Size	Method	Comp. \downarrow	Conv. ↑	Prox. ↑	Stab-shape ↑	Stab-position \downarrow	Stab-sample \downarrow	Ambi. \downarrow
	30×30	Zhou <i>et al</i> .'s method Ours	0.025 0.025	0.997 0.996	385.7 432.0	0.72 0.93	0.225 0.144	147.9 1.2	5.27 3.71
Triple ratio	40×40	Zhou <i>et al.</i> 's method Ours	0.020 0.019	0.996 0.997	364.9 414.3	0.61 0.94	0.282 0.144	432.4 4.8	9.14 6.56
	50×50	Zhou <i>et al.</i> 's method Ours	0.020 0.018	0.996 0.998	350.5 415.7	0.63 0.94	0.241 0.139	1159.8 14.5	15.23 10.10
Perimeter ratio	30×30	Zhou <i>et al</i> .'s method Ours	0.033 0.029	0.920 0.965	369.1 420.0	0.58 0.92	0.261 0.146	163.3 1.3	6.74 3.71
	40×40	Zhou <i>et al</i> .'s method Ours	0.026 0.022	0.914 0.965	345.6 410.6	0.51 0.90	0.288 0.154	453.5 5.2	11.00 6.68
	50×50	Zhou <i>et al</i> .'s method Ours	0.028 0.022	0.897 0.966	315.5 413.2	0.51 0.92	0.269 0.147	1127.0 12.8	20.95 11.23

Table 4: Comparison of Zhou *et al.*'s method and ours in ImageNet-1k. $\uparrow(\downarrow)$ indicates the higher (lower) is better.

Conv. type	Size	Method	Comp. \downarrow	Conv. ↑	Prox. ↑	Stab-shape \uparrow	Stab-position \downarrow	Stab-sample \downarrow	Ambi. \downarrow
	30×30	Zhou <i>et al.</i> 's method Ours	0.024 0.021	0.996 0.996	539.3 565.2	0.61 0.92	0.268 0.104	183.7 0.7	3.47 2.64
Triple ratio	40×40	Zhou <i>et al</i> .'s method Ours	0.018 0.017	0.995 0.997	489.4 530.8	0.51 0.93	0.320 0.094	556.9 0.9	5.84 4.31
	50×50	Zhou <i>et al.</i> 's method Ours	0.019 0.016	0.995 0.998	446.3 492.2	0.48 0.93	0.336 0.099	1561.4 3.7	9.13 7.18
Perimeter ratio	30×30	Zhou <i>et al</i> .'s method Ours	0.030 0.027	0.924 0.961	516.5 553.1	0.50 0.88	0.284 0.126	196.8 0.9	3.41 2.53
	40×40	Zhou <i>et al.</i> 's method Ours	0.024 0.020	0.915 0.965	467.0 526.4	0.43 0.88	0.323 0.114	589.9 2.1	6.42 4.47
	50×50	Zhou <i>et al.</i> 's method Ours	0.024 0.020	0.905 0.966	415.9 491.0	0.43 0.90	0.316 0.118	1436.7 5.4	12.77 7.16

Table 5: Comparison of LAS and ours in CIFAR-100. $\uparrow(\downarrow)$ indicates the higher (lower) is better.

Size	Method	Comp.↓	Conv. ↑	Prox. ↑	Stab-shape ↑	Stab-position \downarrow	Stab-sample \downarrow	Ambi.↓
30×30	LAS	0.039	0.777	646.9	0.40	0.651	398.6	5.78
	Ours	0.021	0.996	618.0	0.92	0.063	0.3	0.97
	Ours with proximity only	0.041	0.767	663.4	0.39	0.506	379.8	8.15
40×40	LAS	0.029	0.801	576.7	0.40	0.606	1209.2	9.49
	Ours	0.016	0.997	566.9	0.94	0.061	0.7	1.61
	Ours with proximity only	0.033	0.763	597.6	0.36	0.551	1229.2	11.72
50×50	LAS	0.028	0.795	528.6	0.38	0.654	3274.7	14.17
	Ours	0.016	0.998	518.9	0.95	0.052	1.7	2.66
	Ours with proximity only	0.034	0.734	541.9	0.35	0.552	3232.9	22.06

Table 6: Comparison of LAS and ours in iNat2021-mini. $\uparrow(\downarrow)$ indicates the higher (lower) is better.

Size	Method	Comp. \downarrow	Conv. ↑	Prox. ↑	Stab-shape \uparrow	Stab-position \downarrow	Stab-sample \downarrow	Ambi. \downarrow
30×30	LAS	0.092	0.445	504.3	0.32	0.491	381.5	63.21
	Ours	0.025	0.996	432.0	0.93	0.144	1.2	3.71
	Ours with proximity only	0.087	0.465	482.5	0.33	0.396	344.5	59.79
40×40	LAS	0.085	0.406	489.8	0.28	0.455	1045.4	111.97
	Ours	0.019	0.997	414.3	0.94	0.144	4.8	6.56
	Ours with proximity only	0.078	0.416	464.1	0.29	0.433	1046.0	99.72
50×50	LAS	0.090	0.383	467.2	0.25	0.476	2928.0	197.73
	Ours	0.018	0.998	415.7	0.94	0.139	14.5	10.10
	Ours with proximity only	0.080	0.401	441.2	0.27	0.433	2827.1	169.30

Table 7: Comparison of LAS and ours in ImageNet-1k. $\uparrow(\downarrow)$ indicates the higher (lower) is better.

Size	Method	Comp. \downarrow	Conv. ↑	Prox. ↑	Stab-shape \uparrow	Stab-position \downarrow	Stab-sample \downarrow	Ambi. \downarrow
30×30	LAS	0.060	0.619	597.9	0.35	0.580	307.6	24.42
	Ours	0.021	0.996	565.2	0.92	0.104	0.7	2.64
	Ours with proximity only	0.062	0.606	603.8	0.36	0.437	284.4	27.62
40×40	LAS	0.051	0.604	564.6	0.33	0.541	938.4	35.78
	Ours	0.017	0.997	530.8	0.93	0.094	0.9	4.31
	Ours with proximity only	0.055	0.580	575.5	0.33	0.501	961.9	42.80
50×50	LAS	0.050	0.601	530.9	0.32	0.590	2591.9	58.28
	Ours	0.016	0.998	492.2	0.93	0.099	3.7	7.18
	Ours with proximity only	0.055	0.570	542.2	0.31	0.497	2594.7	75.64

Table 8: Comparison of DendroMap and ours in CIFAR-100. \uparrow (\downarrow) indicates the higher (lower) is better.

Size	Method	Comp. \downarrow	Conv. \uparrow	Prox. ↑	Stab-shape \uparrow	Stab-position \downarrow	Stab-sample \downarrow	Ambi. \downarrow
30px	DendroMap	0.218	0.624	553.8	0.29	0.334	814.9	2.09
	Ours	0.051	0.943	681.0	0.85	0.080	5.6	0.59
20px	DendroMap	0.055	0.675	404.8	0.34	0.355	2054.1	4.62
	Ours	0.023	0.954	613.7	0.88	0.064	8.4	2.00
16px	DendroMap	0.060	0.600	344.6	0.33	0.317	5950.5	9.90
	Ours	0.017	0.959	596.1	0.90	0.063	12.9	3.11

Table 9: Comparison of DendroMap and ours in iNat2021-mini. $\uparrow(\downarrow)$ indicates the higher (lower) is better.

Size	Method	Comp. \downarrow	Conv. ↑	Prox. ↑	Stab-shape ↑	Stab-position \downarrow	Stab-sample \downarrow	Ambi.↓
30px	DendroMap	0.136	0.430	349.0	0.35	0.270	317.6	7.87
	Ours	0.039	0.959	442.1	0.90	0.144	4.2	3.49
20px	DendroMap	0.150	0.376	268.1	0.22	0.287	1116.0	17.19
	Ours	0.022	0.962	399.8	0.91	0.140	16.9	6.65
16px	DendroMap	0.143	0.336	233.4	0.28	0.306	1970.5	27.49
	Ours	0.022	0.966	405.3	0.92	0.142	23.0	9.60

Table 10: Comparison of DendroMap and ours in ImageNet-1k. $\uparrow(\downarrow)$ indicates the higher (lower) is better.

Size	Method	Comp. \downarrow	Conv. ↑	Prox. ↑	Stab-shape ↑	Stab-position \downarrow	Stab-sample \downarrow	Ambi. \downarrow
30px	DendroMap	0.074	0.643	469.6	0.48	0.324	436.1	4.93
	Ours	0.040	0.960	541.4	0.85	0.117	2.0	1.84
20px	DendroMap	0.069	0.583	361.7	0.40	0.368	447.9	12.19
	Ours	0.029	0.964	488.2	0.89	0.111	2.4	4.73
16px	DendroMap	0.071	0.562	317.3	0.40	0.348	939.2	15.77
	Ours	0.022	0.966	500.0	0.92	0.117	4.5	6.27

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